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On a scaling law for the non-equilibrium relaxation of spin glasses

H Eissfeller and W Kinzel

Institut für Theoretische Physik III, Justus-Liebig-Universität-Giessen, Heinrich-Buff-Ring 16, W-6300 Giessen, Federal Republic of Germany

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Abstract. A scaling relation between remanent magnetization and excess energy of spin-glasses is tested numerically. Based on a simple domain picture of non-equilibrium states of spin-glasses a power law can be derived, which relates the excess energy to the remanent magnetization $\Delta E \propto M^x$. The exponent x is related to the interface exponent y , which determines whether a spin-glass phase exists for $T=0$ or not. Therefore equilibrium properties can be determined by measurement of non-equilibrium quantities. x is calculated numerically for different temperatures T and the spatial dimensions $d=2$ and $d=3$. The numerical results are consistent with the suggested scaling relation.

1. Introduction

Slow decay to thermal equilibrium is one of the most characteristic properties of spin-glasses (Binder and Young 1986, Fischer and Hertz 1991). For example, in thermal equilibrium the magnetization of spin-glasses is zero since the magnetic moments point into random directions. However, after a magnetic field has been applied and then switched off, the spin-glass relaxes into a state with non-zero magnetization M , which slowly decays to zero (Tholence and Tournier 1975). Simultaneously the energy decays to its equilibrium value (Berton *et al* 1984).

A simple but very successful mathematical model for spin-glasses is the Ising model with random bonds (Edwards and Anderson 1975). In fact, numerical simulations show that this model exhibits remanence effects and a slow decay of energy and magnetization in great similarity to real experiments (Kinzel 1979).

But even in simple spin-glass models the relaxational dynamics is not well understood. Even for the model with infinite range of couplings, there is no exact solution of the relaxation far from equilibrium, although a reliable theory of thermal equilibrium exists[†]. Note that even for equilibrium the nature of the excitation is still being debated (Huse and Fisher 1991).

Numerical simulations indicate that at non-zero temperatures this system relaxes to thermal equilibrium, therefore the energy barriers between metastable and equilibrium states are finite (Kinzel 1986). But in numerical simulations the finite size effects are so strong, that even systems up to 10^9 couplings are not sufficient to precisely determine the value of the remanent magnetization for the infinite large system at zero temperature (Spitzner 1991, Kohring and Schreckenberg 1991).

[†] See, however, a recent approach by Horner *et al* 1991.

Some insight into the non-equilibrium dynamics of spin-glasses has been obtained by either exact solutions of phenomenological models (Schreckenberg 1985, Ogielski and Stein 1985) or by scaling relations derived from assumptions on the structure of spin-glass states (Fisher and Huse 1988, Kinzel 1988, Koper and Hilhorst 1988). Both approaches are discussed in Fisher and Hertz (1991).

This paper presents a numerical test of the scaling relation between remanent magnetization and excess energy which was derived by one of us (Kinzel 1988). This scaling relation is based on a very simple ansatz on the structure of the spin-glass state far from equilibrium. As a consequence a relation between excess energy and remanent magnetization is obtained. This scaling relation is checked by numerical simulations. In section 2 the scaling law is explained. The numerical method will be introduced in section 3 and the results are presented in section 4. Finally the results are summarized and discussed.

2. Domain structure of spin glass states

In ferromagnets a state which has been quenched from the paramagnetic to the ferromagnetic phase can be described by domains, which are separately in equilibrium. They grow with time owing to diffusing and annihilating domain walls. The same picture is used for random field problems (Imry and Ma 1975) and is definitely useful to describe one-dimensional spin-glasses at low temperatures (Chen and Ma 1982).

In higher dimensional spin-glasses the structure of non-equilibrium states may be much more complicated. For instance one can imagine a hierarchy of domains inside of domains (Villain 1986), or there may exist a much more ramified structure for which domains are no useful description at all.

In this paper we try to derive consequences of a very simple domain picture, and we check them by Monte Carlo simulations. We assume that after a fast transient relaxation a metastable spin-glass state consists of domains of equilibrium states (see figure 1). This picture should hold at low temperatures, even if there is no spin-glass phase, as long as the thermal correlation length is larger than the typical linear size l of the domains.

l increases with time by diffusion and annihilation of the domain walls. Not much is known about the time dependence of l . Here we do not want to discuss this difficult

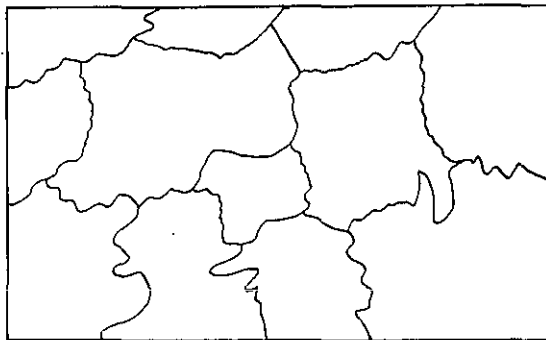


Figure 1. Domain structure of a non-equilibrium spin-glass state. We expect that inside of each domain one of the possible equilibrium configurations is found.

dynamical problem, but rather use the domain picture to derive a relation between remanent magnetization and excess energy.

The energy of such a non-equilibrium state is given by the sum of equilibrium energy (inside of the domains) and the interface energy of the domain walls (as long as no external magnetic field is applied). Hence, if the system has a linear extension L and if d is the spatial dimension one has L^d spins and $(L/l)^d$ interfaces. With each interface having an energy proportional to l^γ , the excess energy per spin can be estimated as

$$\Delta E \propto \frac{1}{L^d} \left(\frac{L}{l} \right)^d l^\gamma = l^{\gamma-d}. \quad (1)$$

In Ising ferromagnets one has $\gamma = d - 1$ i.e. the wall energy is proportional to its surface. In spin-glasses, however, the interface can adjust to the random bonds. The larger the interface, the higher is the probability to find a surface cutting weak bonds. Hence the wall energy increases with its linear extension l weaker than for usual magnets. The power law $\Delta E \propto l^\gamma$ with an exponent $\gamma < d - 1$ has been verified numerically (Bray and Moore 1984, McMillan 1984). Furthermore, scaling arguments at zero temperature show that for $\gamma < 0$ there is no spin-glass phase. In this case the interface energy decreases with the system size and the infinite system is insensitive to its boundaries.

Now we want to estimate the magnetization per spin M from the typical domain size l . Since each domain is in an equilibrium state, its magnetic moments point into random directions. Note that the spins are correlated to their interactions, but for random couplings there is no preferred direction or correlation between the spins. Hence the typical magnetic moment of a domain is proportional to l^{d-2} which is the squareroot of the number of spins.

If in the initial state the spins are ferromagnetically aligned, we expect that after relaxation at least a large fraction of domain moments point into the initial direction. Therefore the total magnetization per spin is estimated as

$$M \propto \frac{1}{L^d} \left(\frac{L}{l} \right)^d l^{d/2} = l^{-d/2}. \quad (2)$$

Equations (1) and (2) are combined to a relation between energy and magnetization. One obtains

$$\Delta E \propto M^x \quad \text{with } x = 2 - \frac{2\gamma}{d}. \quad (3)$$

Note that this relation does not contain the unknown dynamics of $l(i)$. The exponent x can be determined by the exponent γ of the interface energy which has been estimated numerically (Bray and Moore 1984).

The scaling law (3) is a relation between two non-equilibrium quantities. However, from the exponent x one obtains equilibrium properties: namely the value of x gives the interface exponent and only for $x < 2$ the system has a spin-glass phase in thermal equilibrium. Hence the scaling law (3) gives an experimental access to equilibrium properties without obtaining thermal equilibrium.

3. Monte Carlo simulations

We have studied the two- and three-dimensional Ising spin glass by Monte Carlo

simulations (Binder and Young 1986). The Hamiltonian is given by

$$\mathcal{H} = - \sum_{(i,j)n.n.} J_{ij} S_i S_j \quad (4)$$

The spins are described by Ising variables $S_i \in \{+1, -1\}$, and only nearest neighbours on a square and cubic lattice, respectively, are connected by random couplings J_{ij} . We use a Gaussian distribution with zero mean and variance $J = 1$ for the distribution of bonds J_{ij} . For the Monte Carlo algorithm we used the 'Kalos-method' which produces a spin-flip for each computational step (Bortz *et al* 1975, Binder 1979). In figure 2 we compare the efficiency of this algorithm with the one of the standard Metropolis method. Hence, for the three-dimensional spin-glass at half the transition temperature ($T = T_c/2$ with $T_c = (0.9 \pm 0.1)J$ (Bhatt and Young 1988)), we find that the Kalos method is four times faster (this factor depends on system size, implementation, computer, ...).

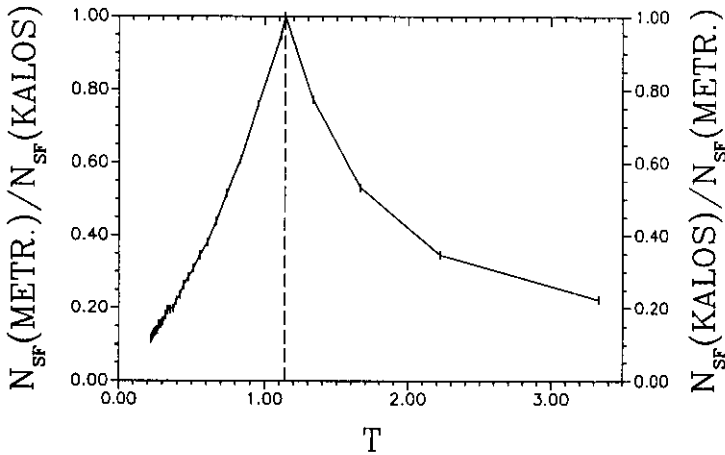


Figure 2. Comparison of algorithm efficiency for the three-dimensional Edwards-Anderson model. The ratio of spin-flip rates, respectively its reciprocal is plotted versus temperature. The vertical line marks the value, where both algorithms have equal efficiency. The simulation was performed for $N = 10^3$ spins, $N_j = 10$ sets of couplings and $t = 3000$ MCS.

We used systems up to $N = 12^3$ spins with helical boundary conditions and averaged the results over up to 24 000 different sets of couplings. The initial state was chosen to be fully aligned $S_i(0) = +1$. Then the system was simulated at a constant temperature T and the magnetization $M(t)$ and energy $E(t)$ per spin were recorded. The time t was measured in Monte Carlo steps per spin (MCS) for the Metropolis algorithm (the Kalos method calculates t from its distribution).

Since we wanted to check the scaling law (3) we first recorded the energy E as a function of M directly. In this case, due to large fluctuations, the energies $E(t)$ recorded for a fixed value of M belong to different times t , for different runs.

It turned out that this sampling method does not work at all. Figure 2 shows the distribution of E and M values. It is a broad cloud which moves only slightly with the time interval of recording. Correspondingly we found a strong dependence of the averaged $E(M)$ values on the system size N and on the length of the recording interval t .

If the energy $E(t)$ and magnetization $M(t)$ are averaged for fixed time t the finite size effects are strongly reduced. Figure 4 shows that both functions can be fitted to a

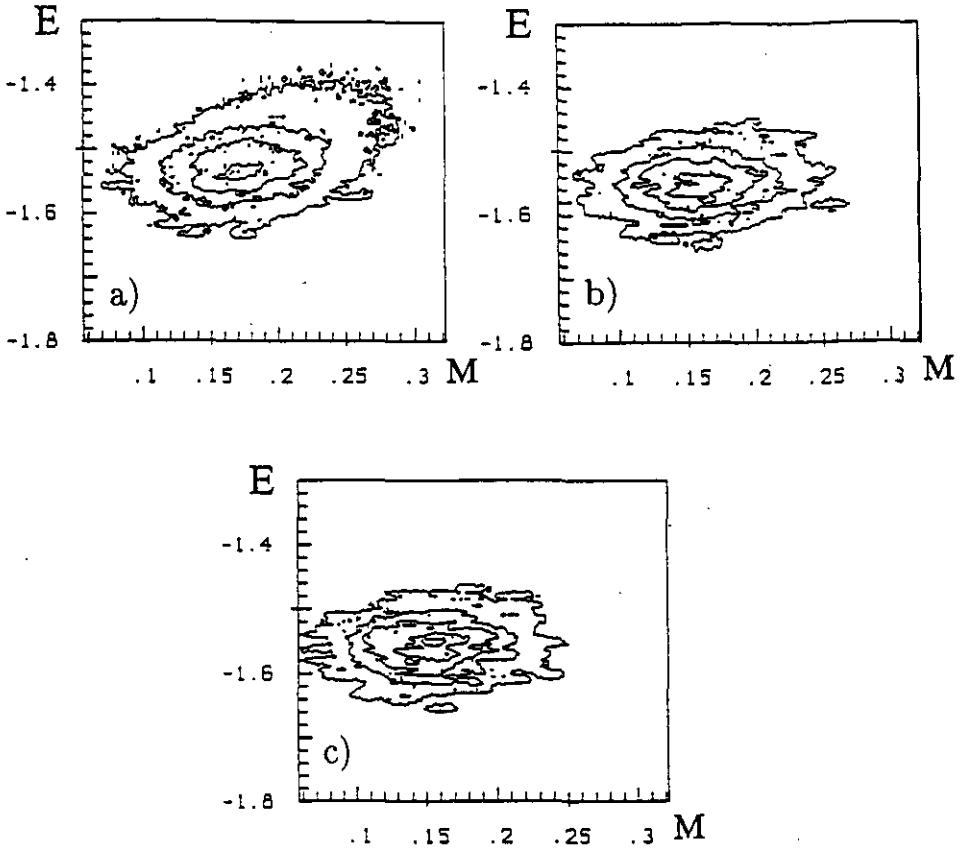


Figure 3. Probability distribution in the energy-magnetization plane, obtained for different times and different samples. Shown are contours of constant probability density. The data are collected from different time intervals (in MCS: (a) $0 < t \leq 1000$; (b) $1000 < t \leq 2000$; (c) $2000 < t \leq 3000$).

power law (Kinzel 1979). The corresponding fits for $d = 2$, which are not shown here are of the same equality.

$$E(t) = E_\infty + Bt^{-b} \quad M(t) = M_\infty + At^{-a} \tag{5}$$

In principle the equilibrium energy E_∞ could have been determined by simulated annealing, but here we used it as a fit parameter. For the infinite system the equilibrium magnetization M_∞ is zero, but in finite systems we expect a value of order $1/\sqrt{N}$.

Figure 5 shows the exponents a and b obtained from fitting the average energy $E(t)$ and magnetization $M(t)$ to the power laws decay (5). The temperature T is rescaled by \sqrt{d} in order to compare the results for different dimensions d , i.e. we define

$$T_s = T/(J\sqrt{d}). \tag{6}$$

We find that the exponents increase with temperature and that the energy decays faster than the magnetization. Both of the exponents are very small, hence the decay is very slow.

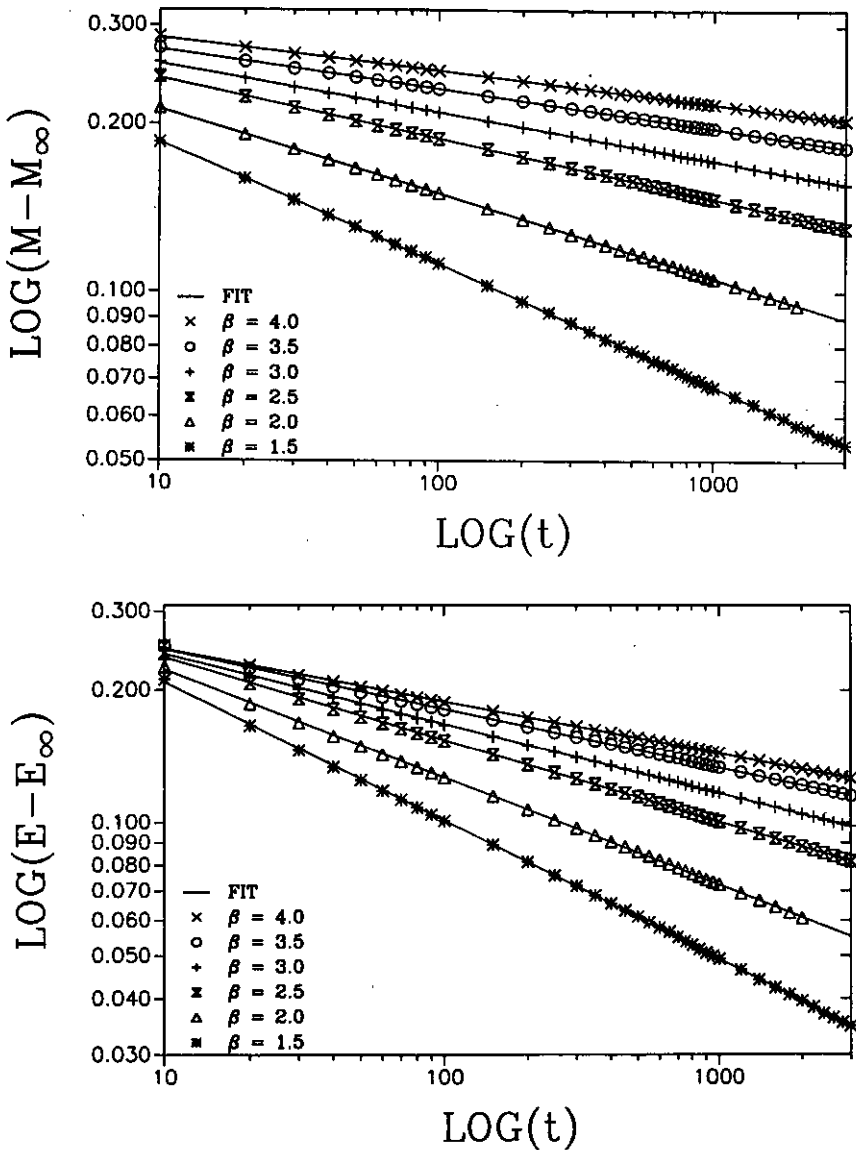


Figure 4. The relaxation of remanent magnetization and excess energy are shown in double logarithmic plots for different values of the reciprocal temperature β and spatial dimension $d = 3$.

4. Scaling exponent

If the power law decay (5) is correct in the thermodynamic limit the energy is a power of the magnetization, as predicted in (3). The exponent x is given by

$$x = b/a. \tag{7}$$

Hence figure 6 showing x as a function of T_s is the main result of this paper. The statistical errors of both a and b are small, but since their absolute values are also

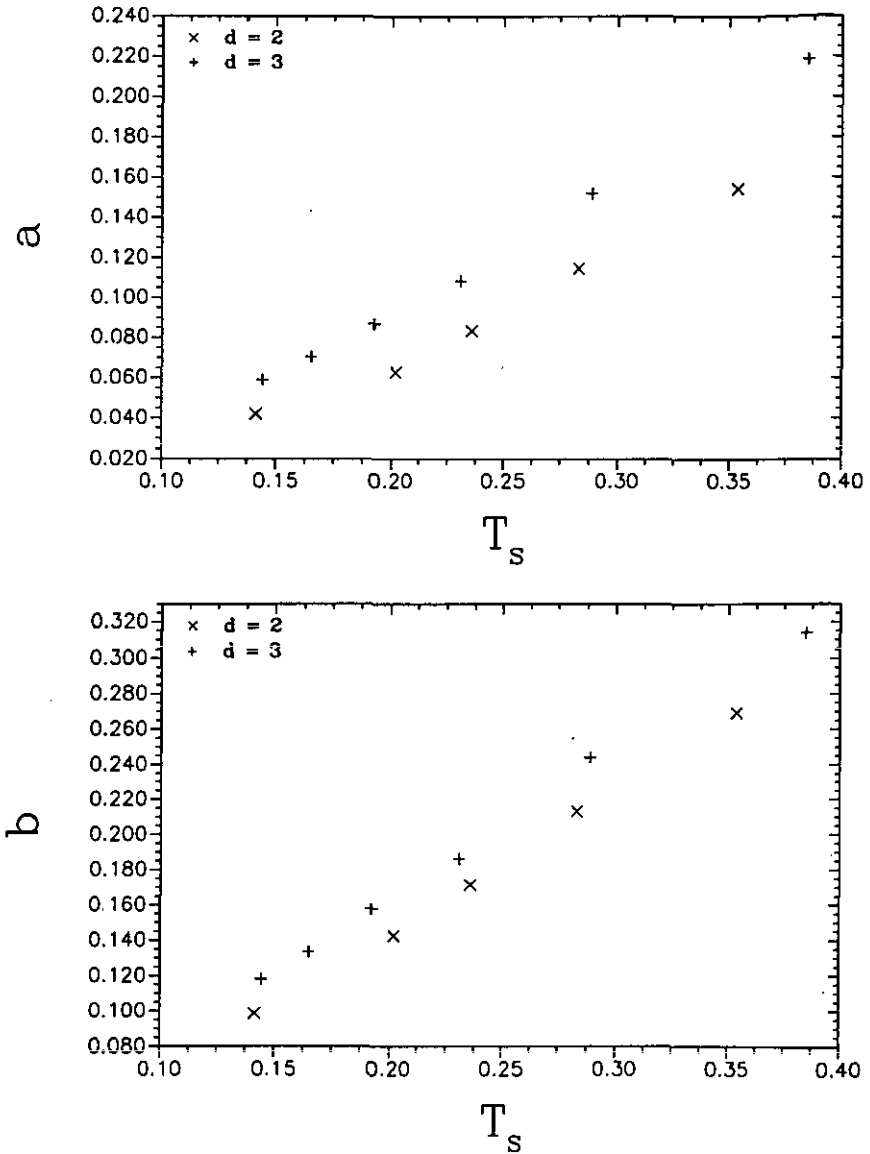


Figure 5. Power law exponents versus scaled temperature. The error bars are smaller than the symbol sizes.

small the statistical error of x is not negligible. Furthermore there are some finite size effects as shown in figure 7. The exponent x obviously decreases with increasing system size. However, the data does not allow a precise extrapolation to the thermodynamic limit.

According to (3) the exponent x is determined by the interface exponent y . For zero temperature, y has been estimated from exact numerical calculations for very small system sizes (Bray and Moore 1984). Using these values of y we obtain $x = 1.9$ ($d = 3$) and $x = 2.3$ ($d = 2$), respectively. Our results for the lowest considered temperature are somewhat larger than these values.

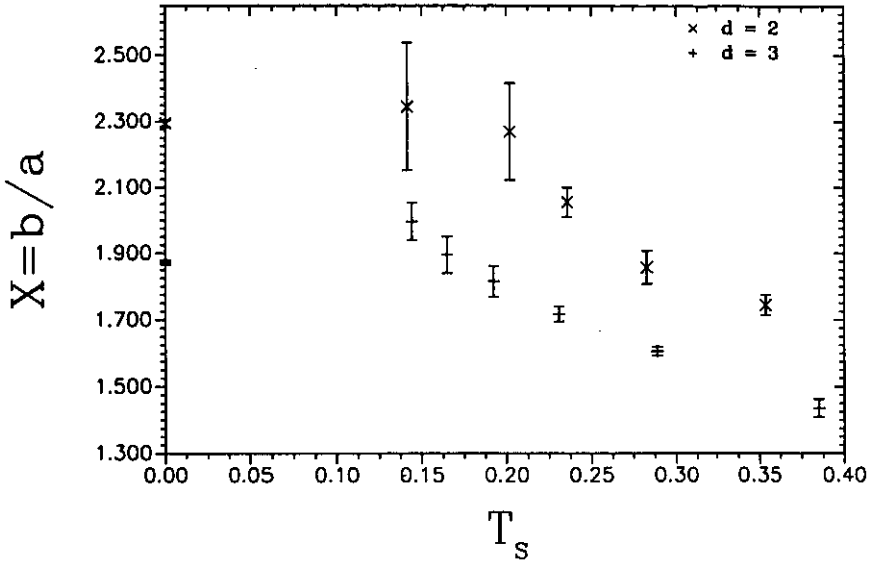


Figure 6. The values and error bars of the exponent x are plotted versus the scaled temperature. d gives the spatial dimension of the model.

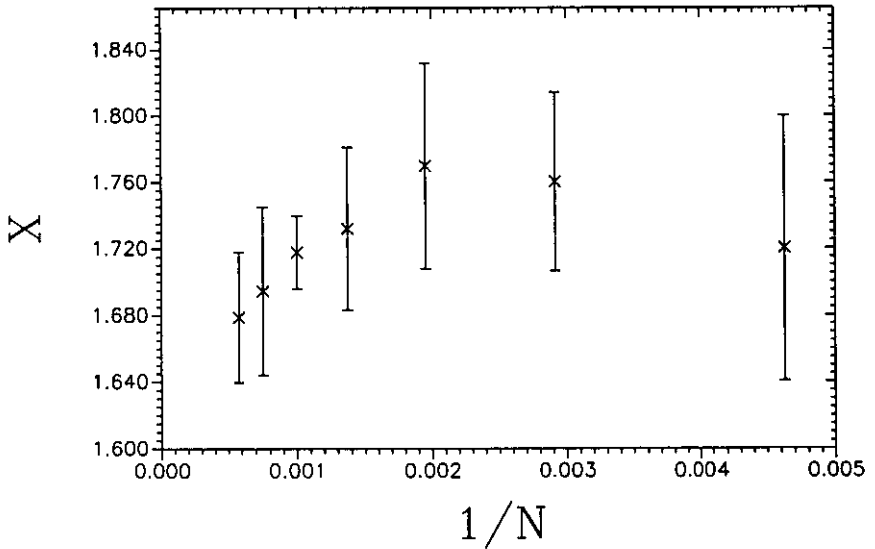


Figure 7. System size dependence of the scaling exponent x .

Fisher and Huse (1986) suggested that at high temperatures the clusters should have a fractal surface with dimensionality d_s . Since the entropy determines the location of the surface, the interface energy scales like a random collection of positive and negative bonds, this would give $y = d_s/2$. In two dimensions d_s is estimated as $d_s = 1.26$ (Bray and Moore 1987) which gives $x = 1.37$. In fact our results of figure 6 do not rule out this possibility, because the data does not allow a reliable extrapolation to high temperatures.

5. Summary and discussion

A simple domain structure is suggested for the spin-glass states far from equilibrium. As a consequence the excess energy depends on the remanent magnetization by a power law with an exponent x , which is related to the scaling exponent y of the interface energy.

If this scaling relation is true, it allows the determination of equilibrium properties from measurements of relaxation far from equilibrium. In particular the system has a phase transition only if the exponent x at zero temperature is smaller than 2.

This scaling relation is tested by numerical simulations of the Edwards-Anderson-model. The energy and magnetization are recorded as functions of time. Each of them can be fitted by a power law decay. As a consequence the energy is a power of the magnetization and the exponent x is calculated as a function of the temperature.

However, a reliable estimate of the value of x was difficult for the following reasons:

- Due to the extremely slow decay of energy and magnetization, the statistical error of x is large.

- There may be a systematic error, since we assume a power law decay. A direct evaluation of energy versus magnetization turned out to be impossible because of strong finite size and finite time effects.

- There is still some dependence of x on the system size. Due to large statistical errors we could not extrapolate x to the thermodynamic limit.

Nevertheless our results show the following behaviour:

- The three-dimensional exponents $x(T_s)$ are lower than the two-dimensional ones.

- $x(T_s)$ is decreasing with increasing temperature T_s .

- At low temperature x is larger than 2 for two dimensions while x is close to the value 2 for three dimensions.

- At low temperature x is consistent with values determined from the interface exponent y at zero temperature. Note that x decreases with the system size N .

Hence, we believe that the numerical results support our scaling theory of the spin-glass states far from thermal equilibrium. Our findings are consistent with the scaling relation, but of course do not uniquely prove it.

There are also experiments on spin glass materials, where both, the decay of energy and magnetization, are recorded simultaneously and fitted to a power law decay (Berton *et al* 1984). Applying our scaling relation (3) their data gives $x = 1.0$ for AuFe 6%, $x = 1.7$ for CuMn 5% and $x = 2.3$ for $\text{Eu}_{0.5}\text{Gd}_{0.5}\text{S}$, respectively. Note that all experiments are performed at finite temperatures $T \neq 0$. This implies that both the metallic materials AuFe and CuMn do have a spin-glass phase, in contrast to the insulator $\text{Eu}_{0.5}\text{Gd}_{0.5}\text{S}$. But more experimental data is needed to check the scaling theory on real systems.

References

- [1] Berton A, Chaussy J, Odin J, Prejean J J and Rammal R 1984 *Solid State Commun.* **49** 959
- [2] Bhatt R N and Young A P 1988 *Phys. Rev. B* **37** 5606
- [3] Binder K *Monte Carlo Methods in Statistical Physics* (Berlin: Springer 1979)
- [4] Binder K and Young A P 1986 *Rev. Mod. Phys.* **58** 801
- [5] Bortz A B, Kalos M H and Lebowitz J L 1975 *J. Comput. Phys.* **17** 10
- [6] Bray A J and Moore M A 1984 *J. Phys. C: Solid State Phys.* **17** L463
- [7] Bray A J and Moore M A 1987 *Phys. Rev. Lett.* **58** 57
- [8] Chen H H and Ma S K 1982 *J. Stat. Phys.* **29** 717

- [9] Edwards S F and Anderson P W 1975 *J. Phys. F: Metal Phys.* **5** 965
- [10] Fisher D S and Huse D A 1986 *Phys. Rev. Lett.* **38** 1601
- [11] Fisher D S and Huse D A 1988 *Phys. Rev. B* **38** 373
- [12] Fischer K H and Hertz J A 1991 *Spin Glasses* (Cambridge: Cambridge University Press)
- [13] Horner H and Freixa-Pascual M 1991 *Z. Phys. B* **80** 95
- [14] Huse D A and Fisher D S 1991 *J. Physique I* **1** 621, 627
- [15] Imry Y and Ma S K 1975 *Phys. Rev. Lett.*, **35** 1399
- [16] Kinzel W 1979 *Phys. Rev. B* **19** 4595
- [17] Kinzel W 1986 *Phys. Rev. B* **33** 5086
- [18] Kinzel W 1988 *J. Phys. C: Solid State Phys.* **21** L381
- [19] Kohring G A and Schreckenberg M 1991 to be published
- [20] Koper G J and Hilhorst H J 1988 *J. Physique* **49** 429
- [21] McMillan W L 1984 *Phys. Rev. B* **29** 4026; 1984 *Phys. Rev. B* **30** 476
- [22] Metropolis N 1953 *J. Chem. Phys.* **21** 1087
- [23] Ogielski A T and Stein D S 1985 *Phys. Rev. Lett.* **55** 1634
- [24] Schreckenberg M 1985 *Z. Phys. B* **60** 483
- [25] Spitzner P 1991 *PhD Thesis* Justus-Liebig-Universität, Giessen
- [26] Thoulence J L and Tournier R 1975 *J. Physique Coll.* **35** C4-229
- [27] Villain J 1986 *Europhys. Lett.* **2** 871